Pumping in a mesoscopic ring with Ahronov-Casher effect

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We investigate parametric pumping of spin and charge currents in a mesoscopic ring interrupted by a tunnel barrier in presence of Aharonov-Casher (AC) effect and Aharonov-Bohm (AB) flux along the axis of the same ring. Generation of a dc current is achieved by tuning the tunnel barrier strength and modulating in time either a radial(transverse) electric field or the magnetic flux. A pure spin current is generated by the interplay of breaking spin reversal symmetry, due to AC effect, and time-reversal symmetry breaking, intrinsic in parametric pumping procedure. We analyze the conditions for operating the AB-AC ring as a pure spin pump useful in spintronics and discuss generalization of our results to Rashba-gate-controlled rings.

The physics of pumping has attracted considerable interest in the last two decades. Parametric pumping of electrons in mesoscopic systems refers to the generation of a dc current by periodic modulations of two or more system parameters (e.g., a gate voltage or a magnetic field) in the absence of a bias voltage. In his original work Thouless[1, 2] studied the integrated particle current on a finite torus produced by a slow variation of the potential and showed that the integral of the current over a period can vary continuously, but must have an integer value in a clean infinite periodic system with full bands. Since then, interest in this phenomenon has shifted to theoretical[3, 4, 5, 6] and experimental[7, 8] investigations of adiabatic pumping through open quantum dots where the realization of the periodic time-dependent potential can be achieved by modulating gate voltages applied to the structure. In recent years parametric pumping has attracted a considerable attention within the emerging field of spintronics[9], as a technique to generate spin-polarized currents in semiconductors[10, 11]. This widens the field of usual magneto-electronics in metals and opens the possibility of combining the rich physics of spin-polarized particles with all the advantages of semiconductor fabrication and technology. Besides, since spin relaxation times of semiconductors can be rather long and coherence of spin states can be maintained up to scales more than 100μm [12], an intriguing question is whether a pure spin current can be pumped in absence of a charge current. This is the case when equal amounts of electrons with opposite spins move in opposite direction in the system. The possibility of generating pure spin currents is attracting enormous interest, not only as a theoretical study *per se*, but also in view of proposed future applications, including scalable devices for quantum information processes[13]. Several works focused on a variety of spin pumps have recently appeared[14, 15, 16, 17, 18].

In this Letter, we report results on quantum pumping through a one-dimensional ring shaped conductor interrupted by a tunnel barrier in presence of time-reversal Aharonov-Casher effect and Aharonov-Bohm magnetic flux and analyze a new scheme of realizing pure spin pumping in mesoscopic systems. An important feature of ring shaped conductors is the appearance of quantum interference effects under the influence of electromagnetic potentials, known as Aharonov-Bohm[19] and Aharonov-Casher[20] effect. The Aharonov-Bohm (AB) phase acquired by a charged particle encircling a magnetic flux is an example of topological phase known for a long time. In 1984, Aharonov and Casher noticed the dual of the AB effect known as AC effect. The AC effect originates from the spin-orbit (SO) coupling between the moving magnetic dipole and the electric field. It is expected to implement the spin current modulation manifested by the AB effect. The AC effect in AB rings with Rashba spin-orbit coupling has been recently investigated both theoretically and experimentally[43]. In numerous studies, the transmission properties of mesoscopic AB and AC rings coupled to current leads were studied under various aspects such as AB flux and coupling dependence of resonances[21, 22], geometric (Berry) phases[23, 24, 25, 26, 27] and spin flip, precession, interference effects[28, 29, 30, 31, 32, 33]. Persistent currents in absence of current leads were studied in Ref.[34]. In our proposal we consider the one-dimensional ring with a time-dependent tunnel barrier and a time oscillating radial (or transverse) electric field in absence of external bias and shall show that dc spin and charge currents are induced by parametric pumping procedure. We calculate analytically the spin dependent transmission and reflection coefficients and apply the scattering matrix approach[35] to calculate the current. Due to the interplay of spin reversal symmetry breaking due to the AC effect and of time reversal symmetry breaking intrinsic in the pumping procedure, a pure spin current can be generated and we shall analyze its behavior in the weak and strong pumping regime.

We consider noninteracting electrons confined to a one-dimensional ring of a radius R with two leads embedded in a radial electric field $\mathbf{E}$. A magnetic flux is assumed to be set along the axis of the same ring. We assume that the length of the ring is smaller than the spin diffusion length, so to neglect spin-flip processes. The ring is interrupted by a single tunnel barrier in one half of the ring, enabling to apply a well defined potential. The one-particle Hamiltonian

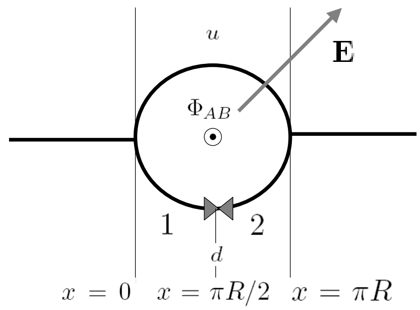

FIG. 1: Aharonov-Bohm-Casher ring. A delta barrier potential (▷◁) is placed in the lower branch of the ring.

of our system is given by $\hat{H} = \hat{H}_R + \hat{V}$, where

$$\hat{H}_R = \frac{1}{2m^\star}(\hat{\mathbf{P}} - \frac{\mu}{2c}\hat{\sigma}\times\mathbf{E} - \frac{e}{c}\mathbf{A})^2, \tag{1}$$

$\hat{\mathbf{P}}$ is the momentum of the electron, c is the velocity of light, $\mu = g\mu_B$ with μ_B being the Bohr magneton, g is the spin g-factor and $m^\star$ is the effective mass of the carriers. The linear term $\hat{\sigma}\cdot(\mathbf{E}\times\hat{\mathbf{P}})$ represents the spin-orbit coupling, $\sigma_i (i = 1, 2, 3)$ are the Pauli matrices; $\mathbf{A} = \frac{\Phi_{AB}}{2\pi R}\mathbf{e}_\phi$ is the vector potential field of the magnetic flux Φ_{AB} with zero magnetic field. $\hat{V}$ is the tunnel barrier potential that we model by a delta function in lower branch of the ring (see Fig.1). To start with, we consider a radial electric field $\mathbf{E} = E\mathbf{e}_r$, with $\mathbf{e}_r$ denoting the unit vector along the radial direction, and put $V = 0$. In the case of a one-dimensional ring, a confining potential $U(r)$ needs to be added in order to force the electron wave function to be localized on the ring. A simple possibility is the harmonic potential $U(r) = \frac{1}{2}K(r-R)^2$, where R is the radius of the ring. Considering only the lowest radial mode[36, 37], the resulting effective 1D Hamiltonian for fixed radius R is:

$$\hat{H}_R^{1D} = \frac{\hbar^2}{2m^\star R^2}(-i\frac{\partial}{\partial\phi} - \frac{\mu E R}{2\hbar c}\sigma_z - \frac{e\Phi_{AB}}{hc})^2. \tag{2}$$

The eigenfunctions of the Hamiltonian are given by $\Psi_{n,\sigma} = 1/\sqrt{2\pi}e^{in\phi}\chi_\sigma$ where $\sigma = \pm$ denotes the spin up and down along the z direction with $\chi_+ \equiv (1,0)^T$, $\chi_- \equiv (0,1)^T$, T denotes the transpose of the matrix. The corresponding eigenvalues are $E_{n,\sigma} = \hbar^2/(2m^\star R^2)(n - \Phi_\sigma/2\pi)^2$ and n is an integer. The phase $\Phi_\sigma = \Phi_{AB} + \sigma\Phi_{AC}$ is the total phase that the electron acquires while the two spin states $\Psi_{n,\sigma}$ move in the ring in the presence of the electric field ($\Phi_{AC} = \pi\theta_0$, $\theta_0 = \frac{\mu E R}{\hbar c}$) and of the magnetic flux (Φ_{AB})[31]. It is instructive to estimate the magnitude of the AC flux in realistic mesoscopic systems. For a ring of radius 10^{-6}m , in external electric field $E \sim 10^4$V/cm, we have $\Phi_{AC}/\Phi_0 \sim 10^{-3}$ (where $\Phi_0 = hc/e$ is the quantum of flux) for particle with gyromagnetic ratio $g \sim 1$. On the other hand, in semiconductors g can be two orders of magnitude larger and the AC flux becomes of the order 10^{-1} which makes the interference effect associated with the AC effect experimentally observable. Assumed that electrons in the two leads are free and have momentum k, the corresponding energy is $\hbar^2k^2/2m^\star$. When an electron moves along the upper arm in the clockwise direction from the input intersection at $x = 0$ (see Fig.1), it acquires a phase $\Phi_\sigma/2$ at the output intersection $x = \pi R$, whereas the electron acquires a phase $-\Phi_\sigma/4$ in the counterclockwise direction along the other arm when moving from $x = 0$ to $x = \pi R/2$ and from $x = \pi R/2$ to $x = \pi R$, respectively. Therefore the total phase is Φ_σ when the electron goes through the loop. The electric field in the ring changes the momenta of the electrons in different spin states $\chi_\pm$ as $k_c^\sigma = k + \Phi_\sigma/2\pi R$ and $k_a^\sigma = k - \Phi_\sigma/2\pi R$, where the subscripts denote the clockwise and counterclockwise direction. The wave functions in the upper(u) and lower(d) arm of the ring can be written as:

$$\Psi_u = \sum_{\sigma=\pm}(c_{u,\sigma}e^{ikx} + d_{u,\sigma}e^{-ikx})e^{i\Phi_\sigma x/2\pi R}\chi_\sigma,$$

$$\Psi_{d\alpha} = \sum_{\sigma=\pm}(c_{d\alpha,\sigma}e^{ikx} + d_{d\alpha,\sigma}e^{-ikx})e^{-i\Phi_\sigma x/2\pi R}\chi_\sigma, \tag{3}$$

where x ranges from 0 to πR and we have assumed that the electron travels in the x direction. The index $d\alpha = 1, 2$ denotes the wave function in the two-halves of the lower branch. The wave function of the electron incident from the left lead in the left and right electrodes is:

$$\psi_L = \psi_i + (r_\uparrow, r_\downarrow)^T e^{-ikx}, \;\; \psi_R = (t_\uparrow, t_\downarrow)^T e^{ikx}, \tag{4}$$

where r_σ and t_σ are the spin-dependent reflection and transmission coefficients, ψ_i is the wave function of the injected electron $\psi_i = e^{ikx}\chi_\sigma$. For an incident electron from the right lead an analogous expansion in terms of reflection and transmission coefficients is possible with i, r_σ (for left lead) and $t_\sigma, 0$ (for right lead) replaced by $0, t'_\sigma$ and r'_σ, i'. This enables us to formulate the scattering matrix equation of the ring system as $\hat{O} = \hat{S}\hat{I}$, where $\hat{O}, \hat{I}$ stand for outgoing and incoming wave coefficients. In the parametric pumping theory, the quantum mechanical current pumped in the ring is related to the derivative of the scattering matrix elements with respect to time-dependent parameters. To calculate the scattering matrix elements we use the local coordinate system[38]. The Griffith boundary conditions[39] state that the wave function is continuous and that the current density is conserved at each intersection. At the point $x = \pi R/2$ in the lower branch the same conditions apply in presence of the delta tunnel barrier. After some algebra we obtain the transmission coefficients $t_\sigma(kR, \Phi_\sigma, z)$ where $z = 2m^\star V/k\hbar^2$ and V is the amplitude of the delta barrier. The explicit expression of $t_\sigma(kR, \Phi_\sigma, z)$ is:

$$\frac{8\,\sin(\frac{\pi kR}{2})\left(-4\,\cos(\frac{\pi kR}{2})\,\cos(\frac{\Phi_\sigma}{2}) + z\,\sin(\frac{\pi kR}{2})\,e^{i\frac{\Phi_\sigma}{2}}\right)}{4\,z\,\cos(\pi kR) - 2\,(5\,i + 2\,z)\,\cos(2\pi kR) + i\,(2 + 8\,\cos(\Phi_\sigma) - 2\,z\,\sin(\pi kR) + (8\,i + 5\,z)\,\sin(2\pi kR))}. \tag{5}$$

In the limit $z \to 0$, $t_\sigma(kR, \Phi_\sigma, z \to 0) = i\cos(\pi kR)\sin(\Phi_\sigma/2)/[\sin^2(\Phi_\sigma/2) - (\cos(\pi kR) - i/2\sin(\pi kR))^2]$. The corresponding transmission probability is $T = |t_\uparrow|^2 + |t_\downarrow|^2$. Our general expression shows that T is independent of the incident spin state. Both the expression of the transmission amplitude and probability are determined by the tunnel barrier strength, the total phase, the kinetic state of the incident electrons, the electric field and the magnetic flux. In the framework of the Landauer-Buttiker theory[40], the quantum mechanical transmission amplitude is related to the conductance. In order to generate a pure spin current in our system, we propose the use of adiabatic quantum pumping[3, 35]. To inject a spin or charge current in the lead 1 one has to modulate two independent (out of phase) parameters of the device in absence of external bias. Adiabatic modulation procedure of the out-of-phase pumping amplitudes dynamically breaks time reversal invariance which in turn leads to a net spin current being pumped. Namely, we adiabatically modulate the Aharonov-Casher flux (i.e. the electric field or the spin-orbit coupling) and the strength of the contact barrier in the lower branch of the ring as: $\Phi_{AC} = \Phi^0_{AC} + \Phi^\omega_{AC}\sin(\omega t + \varphi)$, $z = z_0 + z_\omega \sin(\omega t)$, where φ is the phase difference. In the weak pumping regime, the injected current in a given lead is proportional to the area enclosed in the parameters space, $A_0 = \frac{\sin(\varphi)}{2\pi} z_\omega \Phi^\omega_{AC}$. The $\sin\varphi$ behavior is lost in the strong pumping regime ($\Phi^0_{AC} \ll \Phi^\omega_{AC}$, $z_0 \ll z_\omega$). In the zero temperature limit, the current pumped with arbitrary spin σ in the lead 1 can be derived by the following formula [3]:

$$I_{1\sigma} = \frac{\omega e}{2\pi}\int_0^\tau dt \sum_{l=1,2} \frac{dN_{1\sigma}}{dX_l}\frac{dX_l}{dt}, \tag{6}$$

wherein $\tau = \frac{2\pi}{\omega}$ is the period of the forcing signals, ω is the pumping frequency and e represents the electron charge. The quantity, $\frac{dN_{1\sigma}}{dX_l}$ is the so-called electronic injectivity[41, 42] and is given by:

$$\frac{dN_{1\sigma}}{dX_l} = \frac{1}{2\pi}\Im\{\sum_{j=1,2} \mathcal{S}^{\sigma *}_{1j}\partial_{X_l}\mathcal{S}^\sigma_{1j}\}, \tag{7}$$

with $l = 1, 2$. The pumping parameters are $X_1 \doteq z$ and $X_2 \doteq \Phi_{AC}$, $\Im$ denotes the imaginary part, while $j = 1, 2$ denotes the left lead or right lead. Therefore, the charge and spin currents, I_{ch} and I_{sp}, in the first lead are:

$$I_{ch} = I_{1\uparrow} + I_{1\downarrow}, \;\; I_{sp} = I_{1\uparrow} - I_{1\downarrow}. \tag{8}$$

The dc charge and spin currents generated by the pumping mechanism are shown in Fig.2 as a function of φ in the weak pumping regime for two different values of the AB flux. As shown a *pure* spin current is generated when the AB flux is half-integer. We also find that in the weak pumping regime, the charge and spin pumped per cycle are not quantized. In the strong pumping regime, a pure spin current can be obtained once we properly tune the phase φ.

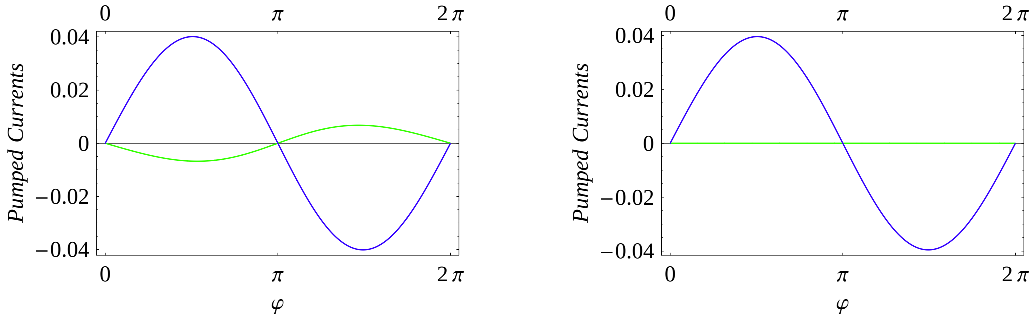

FIG. 2: (Color online) Pumped charge current, green(bright gray) line, and spin current, blue(dark gray) line, in units of $\frac{\omega e}{2\pi}$ as a function of the phase shift φ, in the weak pumping regime. The model parameters in the left panel are: $kR = 10$, $\Phi_{AB} = 0.45$, $\Phi^0_{AC} = 0.1$, $\Phi^\omega_{AC} = 0.1$, $z_0 = 0.1$ and $z_\omega = 0.1$. In the right panel a pure spin current is obtained for $\Phi_{AB} = 0.5$. Fluxes are in units of 2π.

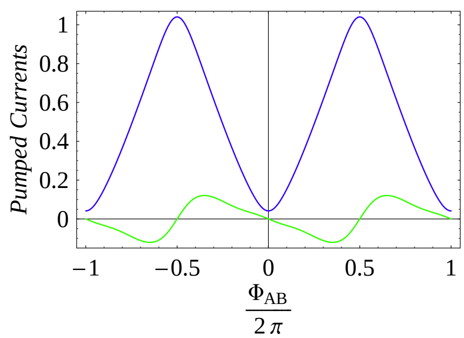

FIG. 3: (Color online) Pumped charge current, green(bright gray) line, and spin current, blue(dark gray) line, in units of $\frac{\omega e}{2\pi}$ as a function of the Aharonov-Bohm flux Φ_{AB}. Parameters: $kR = 10$, $\varphi = 2.6$, $\Phi^0_{AC} = -0.15$, $\Phi^\omega_{AC} = 0.4$, $z_0 = 1$ and $z_\omega = 1.8$.

For specific values of φ, we find that the pumped spin and charge current oscillate with the AB flux and a pure spin current is obtained at half-integer values of the AB flux, as shown in Fig.3.

From the analytical expression of the pumped currents in the weak pumping regime, we find that I_σ is proportional to $\frac{\sigma}{2}A_0\sin(\Phi_{AB} + \sigma\Phi_{AC})$. Thus, the spin current is maximum at half-integer values of the AB flux (in units of 2π) or at integer values of the AC flux, while the charge current is zero. In Fig.4 the pumped spin and charge currents are shown as a function of the pumping amplitude Φ^ω_{AC} from weak to strong pumping regime. We find a finite spin current with a small charge current. The figure also conveys the very important fact that for the entire range the magnitude of pumped spin current increases, which suggests that our model device would pump large spin currents in the very strong pumping regime. Further the pumped charge current is, throughout the range of the pumping amplitude, constant in average. The present analysis also show that Φ^ω_{AC} can be tuned to a regime where the spin transport through the system is quantized, i.e. the average number of spins transmitted in a cycle is integer. This feature allows us to consider the proposed pumping system as a noiseless pump device working in an optimal way.

Since the AC phase can be mapped to the spin-orbit interaction parameter, our results extend to Rashba-gate-controlled rings[43]. In this case, the Rashba electric field results from asymmetric confinement along the z direction perpendicular to the plane of the ring that can be properly tuned by a gate voltage. An effective Hamiltonian analogous to (2) is derived[36], with $-\frac{\mu E R}{\hbar c}\sigma_z$ replaced by $\frac{\alpha m^\star R}{2\hbar^2}\sigma_r$[32], where α represents the Rashba electric field along z and is a tunable quantity. In this case, the AC phase is given by: $\Phi_{AC} = -\pi\sqrt{1 + 2\alpha\frac{m^\star R}{\hbar^2}}$. A complete derivation of eigenvalues and eigenfunctions can be found in Ref.[32], and our expression of transmission and reflection amplitudes is essentially unchanged. For an InGaAs based two-dimensional electron gas, α can be controlled by a gate voltage with typical values in the range (0.5-2.0)$\times 10^{-11}$eVm, which in turn will correspond to values of the AC flux between 0.3π and 3π for the effective mass of InAs $m^\star = 0.023 m_e$ and radius $R = 0.25\mu$m. In conclusion, we have shown a novel scheme of parametric pumping of charge and spin in mesoscopic system. The generation of a pure spin current in AB-AC ring results from the interference effect of electrons with different topological phases and from the adiabatic modulation of two out-of-phase pumping amplitudes that dynamically breaks the time reversal symmetry.

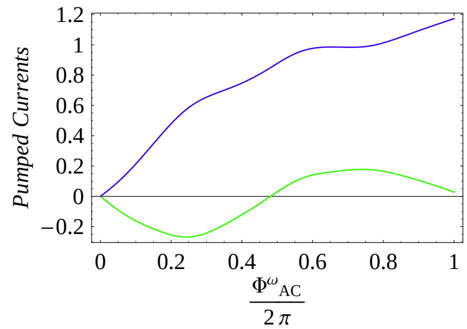

FIG. 4: (Color online) Pumped charge current, green(bright gray) line, and spin current, blue(dark gray) line, in units of $\frac{\omega e}{2\pi}$ as a function of the pumping parameter Φ^{ω}_{AC} for the same parameters of Fig.3.

By changing the applied electric field or the SO coupling, the AC phase contribution to the current oscillation can be tuned to give a pure spin current thanks to spin reversal symmetry breaking. Our proposal is within reach with today's technology for experiments in semiconductor heterostructures with a two dimensional electron gas (2DEG) which has an internal electric field due to an asymmetric quantum well[44] and the spin current in our device could be measurable by the scanning capacitance probe technique. Indeed, spin interference effects in Rashba-gate-controlled ring with a quantum point contact inserted have recently been reported[43].

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